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RESEARCH ARTICLE

Threshold studies for a hot beam superradiant laser including an atomic guiding potential

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Abstract

Motivated by the outstanding short time stability and reliable continuous operation properties of microwave clock masers, intense worldwide efforts target the first implementation of their optical analogues based on narrow optical clock transitions and using laser cooled dilute atomic gases. While as a central line of research large efforts are devoted to create a suitably dense continuous ultracold and optically inverted atom beam source, recent theoretical predictions hint at an alternative implementation using a filtered thermal beam at much higher density. Corresponding numerical studies give encouraging results but the required very high densities are sensitive to beam collimation errors and inhomogeneous shifts. Here we present extensive numerical studies of threshold conditions and the predicted output power of such a superradiant laser involving realistic particle numbers and velocities along the cavity axis. Detailed studies target the threshold scaling as a function of temperature as well as the influence of eliminating the hottest part of the atomic distribution via velocity filtering and the benefits of additional atomic beam guiding. Using a cumulant expansion approach allows us to quantify the significance of atom-atom and atom-field correlations in such configurations. We predict necessary conditions to achieve a certain threshold photon number depending on the atomic temperature and density. In particular, we show that the temperature threshold can be significantly increased by using more atoms. Interestingly, a velocity filter removing very fast atoms has only almost negligible influence despite their phase perturbing properties. On the positive side an additional conservative optical guiding towards cavity mode antinodes leads to significantly lower threshold and higher average photon number. Interestingly we see that higher order atom-field and direct atom-atom quantum correlations play only

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a minor role in the laser dynamics, which is a bit surprising in the superradiant regime.

Keywords
Superradiance, Superradiant Laser, Quantum Optics, Cavity



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Introduction

Ever since the demonstration of the superior operation of an optical atomic clock[1–3] with respect to microwave implementations people started wondering about a possible active clock implementation in form of a superradiant laser[4–20]. In analogy to the very successful hydrogen micro-maser technology[21] such an active device in principle promises at least similar stability and accuracy at a reduced technical cost and fragility. Here an implementation based on a dilute gas of cold clock atoms within a high-Q optical cavity was predicted to be a very promising path to go[5]. As a particular example, model setups based on a beam of inverted ultracold atoms traversing an optical resonator[22–24] were predicted to yield unprecedented stability and accuracy. However, as the operating optical wavelength (~500 nm) is several orders of magnitude shorter than the typical maser lines (~20 cm), the resulting technological challenges are similarly larger.

Already the construction of a suitable beam source is virtually equivalent to operating a continuous atom laser[25,26] and thus poses a way more challenging task than its microwave analog. As the cavity mode volume is also way smaller the atoms have to be tightly confined before sending them through the resonator. Yet another problem is creating atomic inversion with minimal perturbation[27]. Despite intense efforts which recently lead to amazing technical advancements on guided atom beams[28–31] and pulsed superradiant lasing[32–39], a fully continuous operating device[9,19] has not been realized.

In recent calculations it was pointed out that a high precision could already be achieved using a much hotter atomic ensemble with a sufficiently high intra-cavity density[24,40–42]. Here the sheer number of contributing atoms allows to cross the laser threshold inducing an efficient collective synchronization process towards a very narrow effective linewidth. This approach at least in principle requires significantly less components and would allow for a rather compact setup involving only a beam oven, an optical resonator and a pump laser to create inversion. The precise conditions for a concrete setup, however, cannot be reliably analytically predicted and thus require large scale numerical simulations[24]. Here we set up a framework for such simulations which can deal with realistically large atom numbers and velocities. This allows to identify the minimal operating conditions and study the influence of extra elements as velocity filters or guiding potentials.

Methods: Theoretical Model

We consider an ensemble of N two-level atoms with a transition frequency of ω_a coupled to a single mode cavity with resonance frequency ω_c, see Figure 1. The coherent dynamics of this system is governed by the Tavis Cummings Hamiltonian, which in the rotating frame of the atomic transition frequency ω_a reads

$$H=\Delta a^\dagger a+\sum_{j=1}^{N} g(x_j)(a^\dagger \sigma_j^{ge}+\sigma_j^{eg} a), \tag{1}$$

with the atomic transition operator of the j-th atom $\sigma_j^{ge}=|g\rangle_j\langle e|$, the cavity photon creation (annihilation) operator $a^\dagger$ (a), and the cavity-atom detuning $\Delta=\omega_c-\omega_a$. Atoms at a position x along the cavity axis couple to the cavity according to the mode function $g \bullet f(x)=g \bullet \cos(2\pi x/\lambda)$, where g is the maximal cavity coupling strength and λ the wavelength of the cavity photons. The motion of the atoms is treated classically by an initial atomic velocity drawn from a Gaussian distribution with width σ_v corresponding to a temperature T. In the beginning we treat freely moving atoms, later on we investigate the influence of an external optical potential. In both cases the atomic motion is fully determined by the initial velocity and position.

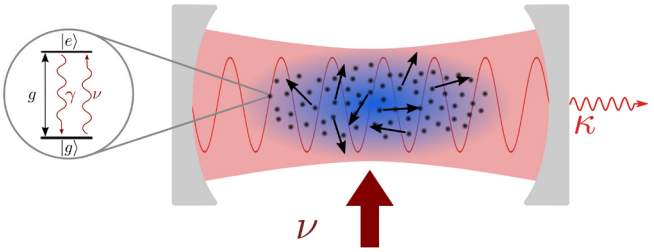


Figure 1. Schematic of the system. Atoms are incoherently pumped with rate ν and couple to an optical cavity. The clusters move according to a Gaussian velocity distribution with standard deviation σ_v, which relates to a certain temperature T. This motion is included along the cavity axis with a position dependent coupling strength g. The atoms decay with the rate γ and the photons leak out of the cavity with a rate κ.

Additional to the coherent dynamics we need to account for the dissipative processes, which are described by Liouvillian term $\mathcal{L}[\rho]$ in the master equation for the system density matrix ρ

$$\dot{\rho}=i[\rho,H]+\mathcal{L}[\rho]. \tag{2}$$

In the Born-Markov approximation the Liouvillian superoperator for a jump operator J_i with a corresponding rate Γ_i can be written in Lindblad form as

$$\mathcal{L}[\rho]=\sum_i \frac{\Gamma_i}{2}\left(2J_i\rho J_i^\dagger-J_i^\dagger J_i\rho-\rho J_i^\dagger J_i\right). \tag{3}$$

The considered dissipative processes are listed in Table 1, including cavity photon losses, individual spontaneous decay as well as incoherent pumping of the two-level atoms.

As we want to simulate systems with up to one million atoms, we cannot solve the full master equation due to exponential scaling of the size of the Hilbert space with the particle number. To this end we employ a mean field approach for operator averages. For a non-explicitly time-dependent operator $\mathcal{O}$ we can calculate the time-evolution of its expectation value as

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\mathcal{O}\rangle=\mathrm{Tr}\{\mathcal{O}\dot{\rho}\}=i\langle[H,\mathcal{O}]\rangle+\left\langle\bar{\mathcal{L}}[\mathcal{O}]\right\rangle \tag{4}$$

with

$$\bar{\mathcal{L}}[\mathcal{O}]=\sum_i \frac{\Gamma_i}{2}\left(2J_i^\dagger\mathcal{O}J_i-J_i^\dagger J_i\mathcal{O}-\mathcal{O}J_i^\dagger J_i\right), \tag{5}$$

where we inserted the master Equation (2) into Equation (4) and used the cyclic property of the trace. To truncate the set of

equations we approximate higher order quantum correlations with the cumulant expansion method[43,44].

The first order approximation (mean field) is sufficient to accurately describe our system (see Appendix), the corresponding set of equations is

$$\langle \dot{a} \rangle = -\left(\frac{\kappa}{2} + i\Delta\right)\langle a \rangle - ig \sum_m f(x_m)\langle \sigma_m^{\mathrm{ge}} \rangle \tag{6a}$$

$$\langle \dot{\sigma}_m^{\mathrm{ge}} \rangle = -\frac{\gamma+\nu}{2}\langle \sigma_m^{\mathrm{ge}} \rangle + 2igf(x_m)(\langle \sigma_m^{\mathrm{ee}} \rangle - \frac{1}{2})\langle a \rangle \tag{6b}$$

$$\langle \dot{\sigma}_m^{\mathrm{ee}} \rangle = (\gamma+\nu)\langle \sigma_m^{\mathrm{ee}} \rangle + igf(x_m)(\langle a \rangle^* \langle \sigma_m^{\mathrm{ge}} \rangle - \langle \sigma_m^{\mathrm{eg}} \rangle^* \langle a \rangle) + \nu, \tag{6c}$$

where the last two equations exist for each atom from $m = 1$ to N. The number of equations in this approximation scales linearly with the atom number N. Still, to simulate a system with several hundred thousand atoms we need further approximations. To this end, we assume that groups of K atoms behave in a sufficiently identical way, such that we can describe them with the same equations. This means, we group atoms into clusters. For the set of equations in 6, this means that the index m now runs only from $m = 1$ to the number of clusters N_{Cl}, but the source term $-ig\sum_m f(x_m)\langle \sigma_m^{\mathrm{gs}} \rangle$ in Equation (6a) gets enhanced by the clustersize K. In the following, we use a clusternumber of $N_{\mathrm{Cl}} = 400$, which is a good compromise between simulating the real setup and still having a feasible computation time (see Appendix).

Table 1. Dissipative Processes. The system features a damped cavity mode, atomic decay and an incoherent pump.

i	c_i	Γ_i	Description
1	a	κ	cavity photon losses
2	σ_j^{ge}	γ	decay from $\|e\rangle_j$ to $\|g\rangle_j$
3	σ_j^{eg}	ν	excitation of the j-th atom

Photon number dependence on temperature

First, we investigate the influence of the atom number N and the temperature T on the steady state photon number. The results in mean field (for a comparison to higher orders see Appendix) are depicted in Figure 2. Unsurprisingly, we always end up with the highest number of photons for the lowest temperature. The drop-off of the photon number for higher temperature can be explained by the Doppler effect, as higher velocities of the individual atoms lead to them being shifted out of resonance. Comparing each curve to the others, we can investigate the significant influence of the atom number on the photon number: Higher atom numbers yield higher photon numbers. While this is not surprising, the crucial aspect of this result becomes apparent, when we normalize the curves to their photon number value for zero temperature (Figure 2(b)): The lacking coincidence of the curves hints to a nonlinear relation between atoms and photons. However, for low atom numbers we see the curves dropping to virtually 0 photons as early as for $\sigma_v/(\lambda\kappa) \approx 0.05$, whereas for higher atom numbers such a temperature has hardly any effect on the photon number. For example, for 10^6 atoms there are still approximately 95% of the photons at zero temperature in the cavity. The red dots indicate the points, at which the photon number has dropped to half their maximal value at zero temperature for each individual curve. These values are depicted in Figure 2(c), they initially follow a quadratic relation, while for higher atom numbers the relation seems linear. From that graph we can clearly see, that using higher atom

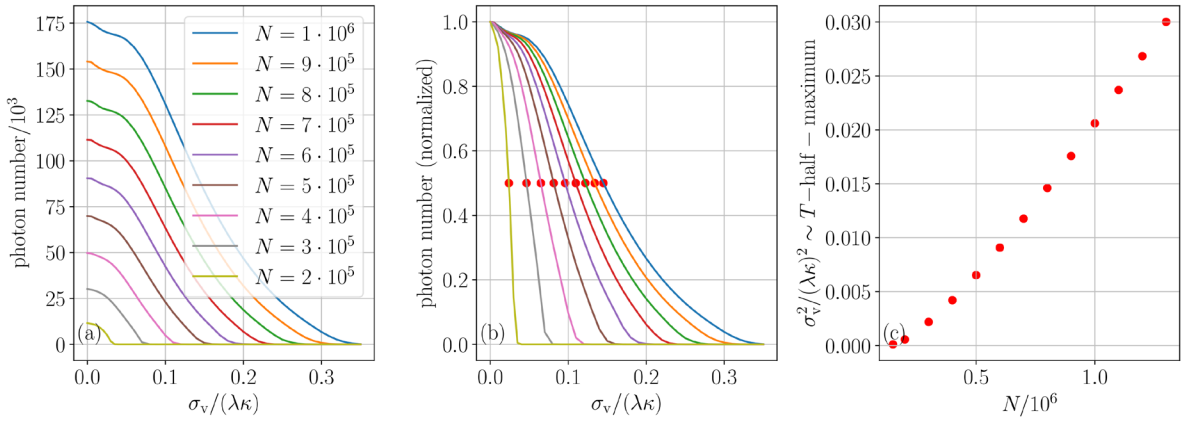


Figure 2. Photon number dependence on temperature and atom number. (**a**) steady state photon number (mean field, see Appendix for comparison to higher orders) as a function of the standard deviation of the velocity distribution σ_v for different atom numbers. (**b**) Photon number normalized to $\sigma_v = 0$ (maximum). The red dots indicate the value at which the curves have dropped to half of their initial value. After normalizing, the curves do not longer align, hinting to a nonlinear relation between atom and photon number. (**c**) Dependence of the threshold value of the velocity spread $\sigma_v^2 \sim T$ on the number of atoms. Larger atom numbers allow for larger possible values of σ_v, while maintaining a lot of photons inside the cavity. The parameters are $\nu = 0.5$, $\Delta = 0$, $g = 0.00136$ and $\kappa = 1$. We neglect γ since it is much smaller than ν, see Equation (6).

numbers allows a higher temperature while still having a decent amount of photons inside the cavity. This could be beneficial to experimental setups, where it is generally difficult to cool the atoms down to low temperature. However, it remains to be seen what this means to the linewidth of the outcoming photons, as higher temperature usually leads to Doppler broadening of the linewidth.

Adding a velocity filter

An interesting idea is to consider a cutoff in the velocity distribution for the atoms. Experimentally, this may be achieved by using a velocity filter for the atomic beam to get rid of the atoms with very high velocity, creating a smaller but colder ensemble. This could be more advantageous than trying to narrow the velocity distribution by cooling, as cooling the atoms proposes an experimental challenge. To simulate such a filter, we start with the same number of atoms as before, but exclude all atoms from the dynamics with velocities higher than the cutoff. We depict the results in Figure 3 for a cutoff at σ_v ($\approx 68\%$ slowest atoms) and at $\sigma_v/2$ ($\approx 38\%$ slowest atoms). To compare, we also depict the previous results without cutoff. In Figure 3(a) the steady state photon number is shown for the different cutoffs. By cutting off the high velocity part of the atomic distribution, we lose only fast atoms, but it always leads to a decrease in photon number. A velocity filter therefore only reduces the photon number. It might still be useful with respect to the linewidth, which is not calculated at this point.

However, the photon number seems to be more stable with increasing temperature, which can be seen in Figure 3(b), where we normalize the graphs to their highest photon number (at zero temperature). While the photon number significantly drops already for low temperature (e.g. $\sigma_v \approx 0.1$) for the uncut case, it stays relatively stable for the other cases, where we exclude the fast atoms.

In summary, including a velocity filter will negatively affect the photon number, but allow it to stay at a relatively stable value for low temperatures. However, it remains to be seen what effect such a velocity filter has on the linewidth of the atoms. In general, one would expect a velocity filter to be beneficial for the linewidth reduction, as the excluded, fast atoms increase the linewidth by means of the Doppler effect.

Dynamics including an optical lattice potential

So far, we assumed that the atoms move freely without any potential. After some time at non-zero temperature, the atoms will be randomly distributed over the mode function. To compare that situation to the zero-temperature case with non-moving atoms, we always place the atoms equidistantly distributed over the mode function. Therefore, the differences in photon number stem from the movement of the atoms itself, not from the somewhat artificial location of atoms on the mode function. The coupling between atoms and cavity field can be controlled (at least for low temperature) by including an optical lattice potential, as the atoms are trapped at the potential minimums. Aligning these potential minimums with the anti-nodes of the mode function leads to enhanced coupling compared to the random sampling of the mode function in the case where atoms move freely. The potential $V(x)$ has the form $V(x) = \Delta E \cos^2\left(\frac{4\pi}{\lambda_{\text{latt}}} x\right)$ with spatial periodicity λ_{latt} and energy barrier ΔE. By using the Hamiltonian formalism one ends up with two linear differential equations of first order for each cluster. These additional equations can simply be added to the equations in the mean field (6) for simulation purposes. To keep the computation time low, we now simulated only $N_{\text{Cl}} = 100$ clusters instead of 400 and only averaged over 10 trajectories instead of 50.

For the optimal case, we assume an optical potential with exactly half the wavelength of the mode function, that way the minima of the optical potential are situated at the maximum values of $|f(x_m)|$. The results of such a simulation are depicted in Figure 4(a). To compare to the previous results, we include the graph that we already calculated, which is the result for an energy barrier of $\Delta E = 0$. The energy is expressed in terms of the recoil energy $E_{\text{rec}} = \frac{\hbar^2 k^2}{2m}$, where $k = 2\pi/\lambda$, v_{rec} is the associated recoil velocity and m is the mass of the atoms. To get exemplary values for these energy scales we use the ^{40}Ca intercombination line between $^1S_0$ and $^3P_1$ with $\lambda = 657$ nm and $\Gamma = 2\pi \cdot 375$ Hz, the cavity linewidth is $\kappa = 2\pi \cdot 120$ kHz. To compare the two energy scales of the energy barrier and the velocity of the atoms, we color the data points according to the fraction of trapped

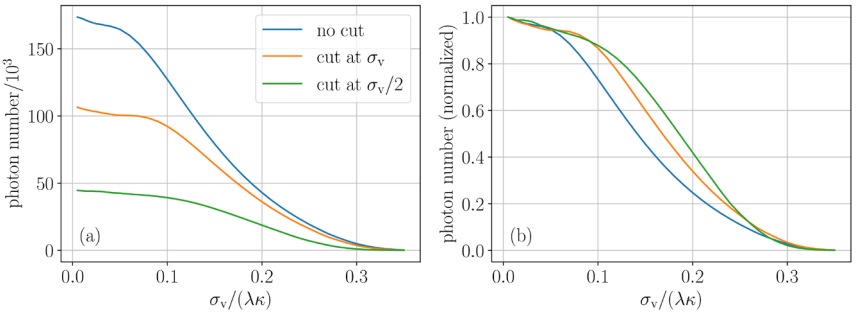


Figure 3. Photon number for different velocity cutoffs. (**a**) Mean steady state photon number. (**b**) Normalized to maximum photon number. Cutting off the the high velocity part of the distribution always yields a smaller steady state photon number, but it remains at a high level for increasing σ_v. We only depict the results in mean field, as the results in mixed order almost perfectly align. The parameters are the same as in Figure 2.

atoms at each particular energy barrier and velocity. For high energy barriers and low velocities we have a lot of trapped atoms, these data points are colored blue, whereas for low energy barriers and high velocities almost no atoms are trapped, these points are colored yellow. As we can see from Figure 4, the results change significantly by including a barrier. Let us first look at the case of low velocities/low temperatures: When the barrier is sufficiently high, then almost all the atoms are trapped at the antinodes of the mode function, leading to high coupling between atoms and cavities and eventually to higher photon number. For higher velocities/temperature, the number of photons stays significantly above the comparison graph for no energy barrier. On the one hand, this is because there are still atoms, that are trapped near the antinode of the mode function. On the other hand, there is a different effect at play, of which we can get a better insight when choosing the wavelength of the lattice slightly different from the wavelength of the mode function. Such a system may be of interest, as there usually is a mismatch between the effective optical lattice periodicity and the optical wavelength.

The results can be seen in Figure 4(b), where we chose $\lambda_{\text{latt}} = 0.99\lambda$. Note, how the results converge for very low temperature, regardless of the value of the energy barrier ΔE. This means, that the coupling is the same as in the barrierless case, where we distributed the atoms equidistantly over the mode function. So by choosing the wavelength of the optical lattice only slightly different to the wavelength of the mode function, we once again sample an average over the mode function, as long as the atoms sit on sufficiently many sites. Therefore the whole advantage of keeping atoms near the antinode of the mode function disappears and the coupling between the atoms and the cavity is the same compared to the coupling in the barrierless case. However, increasing the temperature, we still see a significant difference: Even though the average coupling is the same, the photon number is much higher. The atoms are decelerated, whenever they go over a potential barrier, yielding a reduced average velocity, which in turn decreases the Doppler effect and increases the photon number.

In summary, for low velocities/temperatures, the change in photon number depends mostly on the change of coupling of the atoms, which again depends on the location of the minima of the optical potential compared to the antinodes of the mode function. For higher velocities/temperatures, the other effect of having a lower average velocity due to the barriers comes also into play. The lower average velocity leads to a decreased Doppler effect and therefore a significant increase of photon number.

Conclusions

We studied incoherently pumped two-level-atoms inside a single mode cavity. In particular, we investigated the influence of temperature on the photon number. For high temperatures, where the atoms are fast, the photon number is suppressed because of the Doppler effect. However, we saw that increasing the atom number, to get sufficient collective atom-cavity-coupling, can compensate this suppression. Including a velocity filter leads to a lower amount of atoms participating in the dynamics and therefore, as the number of atoms is a significant parameter, to a lower photon number. We plan to calculate the spectrum to see if excluding the fast atoms has a decreasing effect on the linewidth.

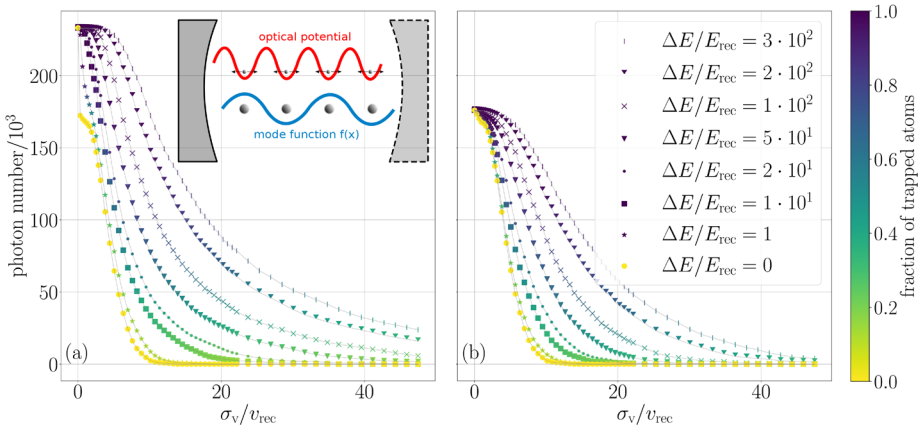


Figure 4. Quasi-steady state photon number including an optical lattice potential with wavelength $\lambda_{\text{latt}} = 0.5\ \lambda$ (**a**) and $\lambda_{\text{latt}} = 0.99\ \lambda$ (**b**) with varying energy barriers. The color of the data points indicates the fraction of trapped atoms, where at blue most of the atoms are trapped, while at yellow most of the atoms can escape their trap. (**a**) $\lambda_{\text{latt}} = 0.5\ \lambda$: Due to the choice of λ_{latt}, for low temperatures most of the atoms are trapped at the antinode of the mode-function, leading to a higher atom-cavity coupling and higher photon number. For higher temperatures, most of the atoms escape their trap, but the photon number stays significantly above the barrierless case. This is due to the energy barriers, which slow the atoms down, yielding lower Doppler effect and therefore higher photon number. (**b**) $\lambda_{\text{latt}} = 0.99\ \lambda$: For low temperatures the results coincide, regardless of the chosen energy barrier. This means, that the atom-cavity coupling is the same as in the barrierless case, where we distributed the atoms equidistantly over a mode function. For higher temperatures, most of the atoms escape their trap, but the photon number behaves similarly to the case with $\lambda_{\text{latt}} = 0.5\ \lambda$. Once again, the atoms get slowed by the barrier, eventually leading to significantly higher photon number compared to the barrierless case. The parameters are the same as in Figure 2.

With an optical lattice potential the coupling between the atoms and the cavity field can be controlled. For low temperatures (compared to the energy barrier), most atoms are trapped and the atom-cavity coupling (and in consequence the photon number) can be engineered. However, for high temperatures, even though most of the atoms are not trapped, the photon number significantly increases compared to the situation without a potential. In some cases, where virtually no photons are generated anymore for a certain σ_v without the potential, we still have a macroscopic photon occupation number, even if σ_v is 4 times as high, by including the optical potential. Therefore including an optical potential becomes very beneficial for higher temperatures.

Appendix. Equations in higher orders

As mentioned in the main text, for our parameter regime the physics is accurately described by mean field. To verify this, we derive the equations for higher orders in the cumulant expansion and compare the numerical results with the mean field. For second order, one derives equations for two-operator averages. These will depend on other averages of two-operator and three-operator products. In order to close the set of differential equations one needs to derive equations for the two-operator averages and approximate the three-operator averages according to 43

$$\langle abc\rangle \approx \langle ab\rangle\langle c\rangle + \langle ac\rangle\langle b\rangle + \langle cb\rangle\langle a\rangle - 2\langle a\rangle\langle b\rangle\langle c\rangle. \tag{7}$$

Using the phase invariance of the system[5,44] one ends up with the set of equations in second order of the cumulant expansion:

$$\langle \dot{a^\dagger a}\rangle = -(\kappa - 2i\Delta)\langle a^\dagger a\rangle + igK\sum_m f(x_m)\left(\langle a\sigma_m^{eg}\rangle - \langle a^\dagger \sigma_m^{ge}\rangle\right) \tag{8a}$$

$$\langle \dot{a^\dagger \sigma_m^{ge}}\rangle = -\left(\frac{\gamma+\kappa+\nu}{2} - i\Delta\right)\langle a\sigma_m^{eg}\rangle + igf(x_m)\left(\langle \sigma_m^{ee}\rangle - \langle a^\dagger a\rangle\right) + ig\sum_{j=1}^{N} f(x_j)\langle \sigma_j^{eg}\sigma_m^{ge}\rangle + 2igf(x_m)\langle a^\dagger a\rangle\langle \sigma_m^{ee}\rangle \tag{8b}$$

$$\langle \dot{\sigma_m^{ee}}\rangle = igf(x_m)\left(\langle a^\dagger \sigma_m^{ge}\rangle - \langle a\sigma_m^{eg}\rangle\right) - (\gamma+\nu)\langle \sigma_m^{ee}\rangle + \nu \tag{8c}$$

$$\langle \dot{\sigma_k^{ge}\sigma_l^{eg}}\rangle = -(\nu+\gamma)\langle \sigma_k^{ge}\sigma_l^{eg}\rangle - igf(x_k)\langle a\sigma_l^{eg}\rangle + igf(x_l)\langle a\sigma_k^{eg}\rangle - \nu\langle \sigma_k^{ge}\sigma_l^{eg}\rangle + 2igf(x_k)\langle \sigma_k^{ee}\rangle\langle a\sigma_j^{eg}\rangle - 2igf(x_l)\langle \sigma_j^{ee}\rangle\langle a\sigma_k^{eg}\rangle. \tag{8d}$$

The advantage of this second order is, that we can include atom-field and atom-atom correlations, however, we pay the price in form of a higher computation time, as the number of equations now scales quadratically with the cluster number instead of linearly as above. The terms involving two atom operators $\langle \sigma_i \sigma_j\rangle$ are responsible for the quadratic scaling, as both i and j are between 1 and N. In an attempt to take the advantage of the upsides of both first order and second order, we approximate two-operator products if they both are atom operators by splitting them, but keep the correlations between atom and field, we call this particular truncation "mixed order":

$$\langle \dot{a}\rangle = -\left(\frac{\kappa}{2} - i\Delta\right)\langle a\rangle - K\cdot ig\sum_m f(x_m)\langle \sigma_m^{ge}\rangle \tag{9a}$$

$$\langle \dot{a^\dagger a}\rangle = -(\kappa - 2i\Delta)\langle a^\dagger a\rangle + igK\sum_m f(x_m)(\langle a\sigma_m^{eg}\rangle - \langle a^\dagger \sigma_m^{ge}) \tag{9b}$$

$$\langle \dot{aa}\rangle = -(\kappa - 2i\Delta)\langle aa\rangle - 2igK\sum_m f(x_m)\langle a\sigma_m^{ge}\rangle \tag{9c}$$

$$\langle \dot{a\sigma_m^{eg}}\rangle = -\left(\frac{\gamma+\kappa+\nu}{2} - i\Delta\right)\langle a\sigma_m^{eg}\rangle + igf(x_m)\langle a^\dagger a\rangle - ig\sum_{j=1}^{N_{Cl}}\sum_{i=1}^{K} f(x_j)\langle \sigma_m^{eg}\sigma_{j,i}^{ge}\rangle - 2igf(x_m)\langle a^\dagger a\sigma_m^{ee}\rangle \tag{9d}$$

$$\langle \dot{a\sigma_m^{ge}}\rangle = -\left(\frac{\gamma+\kappa+\nu}{2} - i\Delta\right)\langle a\sigma_m^{ge}\rangle - igf(x_m)\langle aa\rangle - ig\sum_{j=1}^{N_{Cl}}\sum_{i=1}^{K} f(x_j)\langle \sigma_m^{ge}\sigma_{j,i}^{ge}\rangle + 2igf(x_m)\langle aa\sigma_m^{ee}\rangle \tag{9e}$$

$$\langle \dot{\sigma_m^{ee}}\rangle = igf(x_m)(\langle a^\dagger \sigma_m^{ge}\rangle - \langle a\sigma_m^{eg}\rangle) - (\gamma+\nu)\langle \sigma_m^{ee}\rangle + \nu \tag{9f}$$

$$\langle \dot{\sigma_m^{ge}}\rangle = -\frac{\gamma+\nu}{2}\langle \sigma_m^{ge}\rangle - igf(x_m)\langle a\rangle + 2igf(x_m)\langle a\sigma_m^{ee}\rangle \tag{9g}$$

$$\langle \dot{a\sigma_m^{ee}}\rangle = -(\gamma+\nu)\langle a\sigma_m^{ee}\rangle - \frac{\kappa}{2}\left(a\sigma_m^{ee}\right) + \nu\langle a\rangle - igf(x_m)\langle aa\sigma_m^{eg}\rangle - ig\sum_{j=1}^{N_{Cl}}\sum_{i=1}^{K} f(x_m)\langle \sigma_m^{ee}\sigma_{j,i}^{ge}\rangle + igf(x_j)\langle a^\dagger a\sigma_m^{ge}\rangle. \tag{9h}$$

For the sake of better readability we refrain from carrying out all the expansions of three-operator expectation values like $\langle aa\sigma_m^{ee}\rangle$. In order to end up with a closed set of equations, one needs to approximate the three-operator expectation values by two-operator expectation values according to Equation (7). By neglecting the atom-atom correlations we get rid of the quadratic scaling, but we are still able to keep higher order correlations, namely the ones between atom and field. Grouping the atoms into clusters is more involved in the second order and mixed order as opposed to first order, one has to carefully take into account for example the correlations within a cluster[17,45].

Determining the steady state photon number

In order to study the photon number values depending on different parameters we initialize the set of differential equations with a certain parameter set and let the system evolve until it reaches its quasi-steady-state. The velocities are drawn from a Gaussian distribution with width σ_v corresponding to a temperature. We average over the photon number from the timepoint, at which the quasi-steady-state has been reached, see Figure 5. Using more clusters leads to much smaller fluctuations, however, this also requires more computation time. Finding a compromise between high clusternumbers and reasonable computation time is discussed in the next section. Moreover, one can imagine that certain samples of the distribution exhibit special properties, in order to average them out we use 50 (unless indicated otherwise) different samples and also average over them.

Comparison expansion orders and number of clusters

Limited computation time forces us to make compromises, as both higher expansion orders and higher number of clusters describe the physical reality better, but also require more computation time.

In the following we compare the results for different expansion orders and number of cluster. In Figure 6(a) the solid

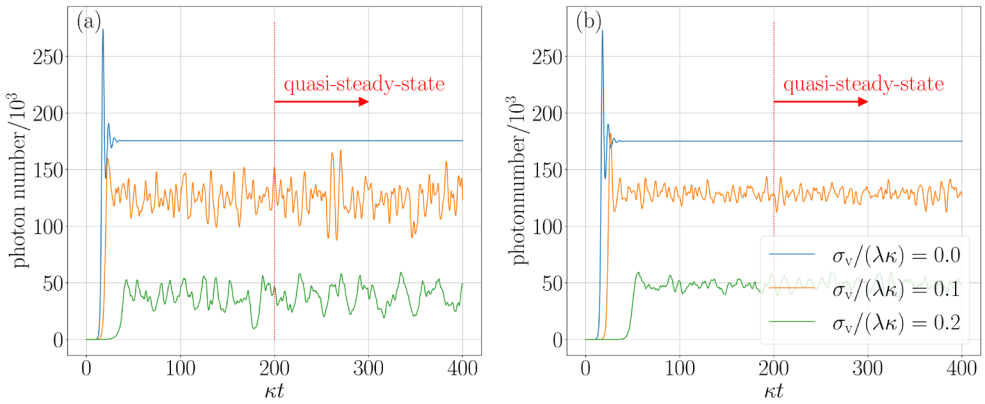


Figure 5. Determining the steady state photon number. Mean photon number time evolution for different σ_v with (**a**) 200 atom clusters and (**b**) 1000 atom clusters. After a certain time, the system reaches its quasi steady state. The fluctuations become smaller if we divide the velocity distribution into a larger number of atom clusters. The other parameters are the same as in Figure 2 for $N = 10^6$.

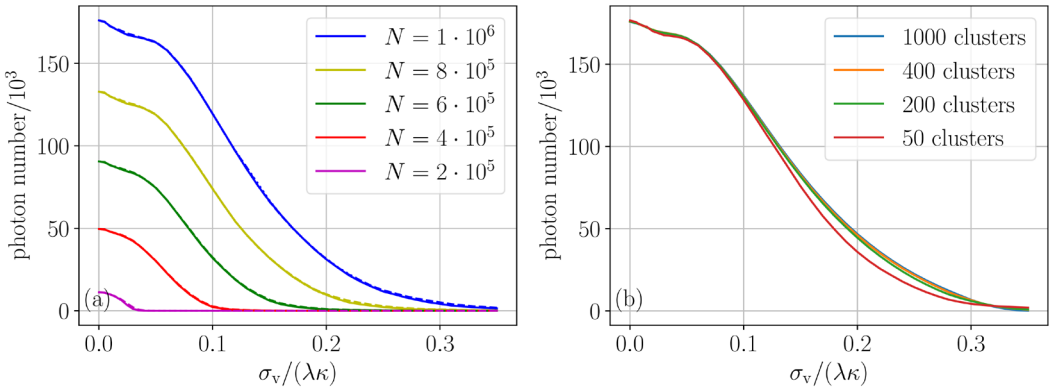


Figure 6. Expansion orders and number of clusters. (**a**) The solid line represents mean field, the dotted line is mixed order and the dashed line depicts the result in second order, we use $N_{Cl} = 25$. The results for all orders agree very well. (**b**) The chosen number of clusters N_{Cl} makes a difference, especially for medium temperatures. However, increasing the number of clusters from $N_{Cl} \approx 400$ has only a marginal effect on the results. The other parameters are the same as in Figure 2 for $N = 10^6$.

line depicts the first order, the dotted line mixed order and the dashed line represents the data in second order. We see that all three lines almost perfectly align. Therefore, using higher expansion orders does not lead to significant differences and the first order approximation captures the essential physics in our parameter regime.

In Figure 6(b) we depict the results in first order for different clusternumbers ranging from $N_{Cl} = 50$ to $N_{Cl} = 1000$. Here we see, that varying the number of clusters N_{Cl} does have an influence on the results. We chose $N_{Cl} = 400$, as the difference between choosing $N_{Cl} = 400$ and as high as $N_{Cl} = 1000$ is only marginal, while the computation time significantly increases.

Data availability

Underlying data

Figshare: Superradiant_laser_Figures. https://doi.org/10.6084/m9.figshare.c.6781920.v1[46].

This project contains the following underlying data:

- Data used in Figures 2-6. All data are stored using DelimitedFiles.jl package in Julia.
- Data are stored together with the source code (see below), specific files to readout and plot the data are provided.

Data are available under the terms of the Creative Commons Zero "No rights reserved" data waiver (CC0 1.0 Public domain dedication).

Software availability

- Source code available from: https://github.com/martinf97/SRL_paper
- Archived source code at time of publication: https://doi.org/10.5281/zenodo.8232295[47]
- License: Creative Commons Zero "No rights reserved" data waiver (CC0 1.0 Public domain dedication).

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Current Peer Review Status: ? ? ? ?

Version 1

Reviewer Report 29 February 2024

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?

Tiantian Shi
Peking University, Beijing, China

This paper describes theoretical threshold conditions and the predicted output power of hot beam superradiant laser by numerical simulation of larger atomic numbers and velocities in reality. The paper explains the significance of atom-atom and atom-field correlations. Active optical clock (superradiant laser) has received increasing attention.

In general, I find this work useful, interesting, and suitable for a broad physics audience. Generating stable lasers using hot atomic beam is a hot topic, and this paper adds important depth to this field. It is novel and rigorous enough for approval, provided some points are addressed.

1. A number of small problems exist in the text, figure. 1) While these findings are indeed interesting, I suggest the authors alter their wording in consecutive sentences in the abstract. 2) “In the following, we use a clusternumber of NCl = 400”. In this sentence, a space is missing between "cluster" and "number". 3) For Figure 4, it can be transformed into a three-dimensional diagram for readers to see more clearly. Suggest the author to add an axis based on the independent variable of optical lattice depth. For example, the x-axis represents velocity, the y-axis represents lattice depth, and the z-axis represents the number of photons.

2. In Figs. 2(a) and 2(b), the x-axis is sigma_v/lambda_kappa, while in Fig. 2(c), the y-axis is the square of sigma_v/lambda_kappa (which corresponds to temperature). Since the relation between the FWHM sigma_v^2/lambda_kappa^2 and atom number N is initially quadratic, I think it would be interesting to plot the FWHM sigma_v/lambda_kappa against N for small N, so as to confirm the quadratic relation. Also, the cutoff temperature in Fig. 2(a) may also have interesting behavior.

3. The velocity of thermal atoms contributes to decoherence and broadening in different ways: the parallel velocity (along the atomic beam direction, perpendicular to the axis of the optical cavity, I assume) affects the cavity-atom interaction time and introduces second-order Doppler shifts, whereas the transverse velocity (i.e. perpendicular to the beam direction and along the cavity axis) mainly contributes via first-order Doppler effect. It is also worth noting that the velocity

distributions along and perpendicular to the atomic beam direction scale differently with temperature. I think the authors should put more analysis into this point, preferably in another Appendix section.

4. The continuous-wave superradiant laser can also be realized by pumping the thermal atoms inside a vapor cell placed in a low-finesse cavity. As far as I know, the laser linewidth of active optical clock is measured as narrow as 53 Hz utilizing the dual-wavelength good-bad cavity technique. Detailed data can be found in this article: Opt. Express 27, 22040 (2019).

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Is the work clearly and accurately presented and does it cite the current literature?
Yes

Is the study design appropriate and does the work have academic merit?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Yes

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Atomic clock; Active optical clock; Superradiant laser

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 20 February 2024

https://doi.org/10.21956/openreseurope.17820.r37523

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?

Stuart J Masson
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Silvia Cardenas-Lopez
Department of Physics, Columbia University, New york, New york, USA

This manuscript analyzes the steady-state collective emission of a thermal cloud of atoms into a cavity. The authors study the impact of temperature on the number of cavity photons by modelling the atomic motion classically and using mean field equations to describe the atom-cavity interaction. Due to motion, temperature decreases the number of cavity photons, although the effect is mitigated as the number of atoms is increased. Additionally, numerical simulations demonstrate that incorporating a velocity filter leads to a reduction in photon count but greater stability to increasing temperature, while the introduction of a trapping potential reduces negative effects caused by the temperature.

The characterization of the impact of temperature on the photon number presented in the manuscript complements other works considering similar systems (Ref. [36] of current version of the manuscript), and the work makes a useful contribution to a topical field. The manuscript is in general clearly written and the discussion of methods is detailed. Our primary concern is that the analysis is not thorough enough. In particular, the lack of discussion of the linewidth of the emitted light brings into question the description of the system as a superradiant laser. As discussed in Ref. [36], the Doppler-broadened atomic spectrum has a full width at half maximum (FWHM) that can be much larger than the linewidth of ultranarrow transitions. It is possible that the system could be out of the bad cavity regime, which would increase the cavity pulling effects. Hence, it would seem to us that a thermal cloud emitting into a cavity cannot be called a superradiant laser for arbitrarily large temperatures, and that the analysis of cavity photon number must be complemented with a study of the linewidth before concluding that the system indeed realizes a superradiant laser. An analysis of the spectrum or more careful discussion of the possible impact of temperature on the linewidth would greatly enhance the message of the manuscript.

Some additional comments and questions are itemized below.

1. An expanded discussion on the cluster approximation used to simplify the analysis would be beneficial. By what metric are the atoms clustered? Are the clusters of equal size? When is the approximation valid? Alternatively, it would be helpful to cite references that cover this aspect.

2. Is there any intuition behind the fact that all the behavior appears to be well captured by mean field? At least for the conventional superradiant laser with motionless atoms (Ref. [5] for example), the threshold is identified as a change in sign in the two-particle correlation between different atoms (Eq. 6 of Ref. [5]), so using second order cumulant expansion is necessary.

3. Why is the laser threshold defined as in Fig. 2? It would seem that it is the velocity spread at which the photon number drops by half, which seems arbitrary. Given the specific use of the term threshold in laser physics, perhaps an alternative word could be used.

4. The analysis is currently performed for a narrow range of parameters. How does the threshold change when the velocity spread is fixed, but other parameters like the pumping rate, or the

cavity coupling are changed?

5. In order to facilitate comparison with experiments, it would be useful to convert, either in the text of figure caption, the velocity spread to a temperature. It would be useful to understand the threshold as specific target temperature ranges for relevant atomic species such as calcium and strontium.

6. It is mentioned that the number of cavity photons with guiding potential is much larger than in a system with no potential at the same temperature. How does this effect scale with the number of atoms?

7. We are having some trouble understanding the set of equations 6a-c. In Eq. 6c, taking the conjugate of the raising operator expectation value gives the lowering operator, so we believe it should either be the expectation value of the conjugate of the lowering operator, or simply the expectation value of the raising operator. We also believe there is a minus sign missing from the first term in Eq. 6c. There also does not appear to be any terms relating to the Doppler shift.

8. Why is the temperature distribution of the atoms modelled by a Gaussian distribution rather than a Maxwell-Boltzmann distribution?

9. The language used in some parts of the paper is not precise. In particular, in the introduction, in several places vague comparisons are made (e.g., "way more"), where a more scientific discussion of the differences would strengthen the motivation.

Is the work clearly and accurately presented and does it cite the current literature?
Yes

Is the study design appropriate and does the work have academic merit?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Partly

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Quantum optics, cavity QED, superradiance

We confirm that we have read this submission and believe that we have an appropriate level

of expertise to confirm that it is of an acceptable scientific standard, however we have significant reservations, as outlined above.

Reviewer Report 01 February 2024

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?

Dylan Young
University of Colorado Boulder, Boulder, Colorado, USA

In this work, Fasser *et al.* develop a numerical framework for simulating the dynamics of thermal atoms exhibiting continuous superradiance when coupled to an optical cavity. Using this framework, they analyze the steady-state output power of such a superradiant laser (expressed as the number of photons in the cavity) over a range of different atom numbers and temperatures. They also introduce a couple of intriguing thought experiments, studying how the laser's behavior changes when modifying the velocity distribution of the atoms (the "velocity filter") or encouraging the atoms to move towards regions of large cavity coupling (the "atomic guiding potential").

The system studied in this article has received considerable interest in recent years as a potential low-complexity realization of a superradiant laser, making this work both relevant and timely. The numerical framework introduced appears to be quite successful at capturing the physics, and it should be straightforward to use this model to simulate a real system. However, there are a few important points that I believe need to be addressed or clarified, so I approve this article with reservations. My comments are below:

1. As presented, the Hamiltonian in (1) appears to only consider 1D motion of the atoms, namely along the cavity axis. However, as far as I'm aware, the idea of a "hot beam superradiant laser" typically involves shooting a thermal beam of atoms transversely through the cavity, giving the atoms a finite transit time τ. Currently, this point does not seem to be raised at all in the manuscript. The motivational work cited by the authors [24, 40-42] all appear to involve such a transverse atomic beam, making this a confusing omission. Some clarification here would be appreciated.
    - If the authors are not trying to simulate such a transverse beam, it would help to explicitly state how their model differs from a transverse beam system in order to avoid confusion. In this case, they should also provide a potential experimental realization of the proposed model.
    - If this model is meant to be a limiting case (such as letting $\tau \to \infty$), the authors should either justify why this regime is valid or clarify the limitations of this regime.
2. On a related note, was the optical pumping rate ν introduced to preclude the need to model such a transverse beam? If so, is it clear that the qualitative behavior of the system should be the same? It's worth noting that even if the atoms are confined to the cavity via some attractive potential, optical pumping will eventually heat up and kick out the atoms

from the cavity (unless additional cooling is performed), limiting the timescale over which steady-state behavior can be observed.

3. The authors should explain how they chose the parameters used in their simulations (which I believe are listed in the caption of Fig. 2), especially since they want to simulate "realistic particle numbers and velocities."
4. The discussion of the photon number dependence on temperature and atom number could be enriched by the addition of intuition/scaling arguments. For example, the authors note that "the relation [between half-maximum temperature T and atom number N] seems linear" for large N. Is this expected behavior?
5. In the context of superradiance, the word "threshold" is often used in terms of the threshold for lasing, i.e. the point where the photon number goes from roughly 0 to positive. It doesn't seem to be used in this way here. Could the authors define what they mean by "threshold?"
6. On that note, if you look at the scaling of the "threshold atom number" (as defined in point 5) with temperature in Fig. 2a, it seems roughly linear, which would be a different scaling than the half-max temperature shown in 2c. If true, that could be an interesting observation.
7. In the velocity filter section, again I think it would benefit the paper to connect the results of the simulation to expectations. Can the stable photon number for low temperatures be explained by the fact that the velocity filter removes all atoms that are Doppler shifted out of resonance?
8. In the optical lattice potential section, presumably the authors are discussing a repulsive potential rather than an attractive potential? (If it were attractive, I would be concerned that the atoms would speed up instead of slow down when approaching the potential.) If so, it would help to specify this.
    - This point is especially relevant if you consider atoms approaching the cavity from the side in a hot beam (i.e. they start outside the potential). In this case, you could also ask if a repulsive potential might limit the flux of atoms into the cavity mode in addition to slowing them down.
9. Finally, a minor point: in Fig. 4, why does the cartoon picture of the cavity have one solid mirror and one dashed mirror? Maybe I missed the explanation, but I'm not sure what it adds.

Is the work clearly and accurately presented and does it cite the current literature?
Yes

Is the study design appropriate and does the work have academic merit?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Superradiance, cavity QED, collective atom-light interactions, out-of-equilibrium atom-cavity physics

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 31 January 2024

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?

Fabian Finger
ETH Zurich, Quantum Center & Institute for Quantum Electronics, Zürich, Switzerland

The paper “Threshold studies for a hot beam superradiant laser including an atomic guiding potential” analyzes a superradiant laser with incoherently pumped two-level atoms in an optical cavity. It specifically addresses scenarios with realistic atom numbers where atom clusters move classically according to a Gaussian velocity distribution. The present study incorporates both cavity and atomic decay, employing a mean-field approach alongside a higher-order cumulant expansion for comparison. The authors observe that steady-state photon numbers increase with atom number but decrease with rising temperature of the atomic sample. However, higher atom numbers can mitigate the adverse effects of increased temperatures. They also explore using a velocity filter instead of cooling methods to effectively reduce the atomic sample's temperature. This technique decreases the steady-state photon number due to a reduction in atoms, but it also leads to a less steep decline in photon numbers as a function of temperature. Further, the study examines the influence of atomic confinement via an optical lattice, finding that trapping atoms at the antinodes can maintain high steady-state photon numbers at higher temperatures, especially with strong trapping potentials. Another advantage of the lattice is the reduced Doppler shift, tested by varying the lattice wavelength. The authors conclude that finite barriers in the lattice reduce average velocity and associated Doppler shifts, thereby increasing photon numbers, a factor more significant at higher temperatures. They specifically mention that their study only encompasses the steady-state photon number and state that the linewidth of this system, a key parameter for superradiant lasers, is subject to further investigation.

Overall, I think the manuscript is well-written, interesting both from an experimental and theoretical perspective and addresses a very timely subject. I approve it with some comments and questions that hopefully will improve the clarity of the manuscript:

- The term 'threshold' in the text is somewhat ambiguous. I assume the authors mean a

specific threshold photon number, but this could be confused with a lasing threshold. The clarity of the manuscript could be improved by clearly defining the threshold in the main text.
- Could the authors clarify the dimensionality of their calculations? The position-dependent coupling strength g(x) includes atomic motion along the cavity axis, which suggests 1D. However, where does the velocity of the beam oven enter? Or is it neglected in the present study?
- The term “atomic guiding potential” suggests the notion of an atomic beam guided by, e.g., an optical dipole trap, which, however, does not seem to be the case here. The authors might consider renaming it specifically to clarify that the influence of an additional intra-cavity optical lattice is studied.
- I think it would be helpful if the authors could expand the discussion of the incoherent pump in the main text. I assume the pump term already includes the use of an auxiliary third level with fast decay to the excited state, which is then adiabatically eliminated, but it would make it easier to follow the calculations.
- A brief comparative discussion of this work with the study mentioned in Ref. 24, which also analyzes a superradiant laser using realistic atom numbers, would enrich the introduction I think.
- The authors use specific parameters (atom number, coupling strength, pump strength, cavity leakage, etc.) for all their calculations. Are those numbers related to an existing implementation of a superradiant laser? If not, how do they compare to existing setups? It would be great if the authors could briefly discuss those parameters in the manuscript's main text, similar to the 40Ca example they give.
- Could the authors comment on which other effects could lead to line broadening except for the Doppler effect? How important are they, and why are they neglected?
- Can the authors comment briefly on how to implement velocity filters, especially if they are easier than cooling methods?
- One important aspect not discussed in the manuscript is atom loss and heating effects (light-assisted collisions, photon recoil), for example, through the incoherent pump, large photon numbers in the cavity, and the 1D optical lattice to confine the atoms and potentially maximize the coupling. Do the authors have any estimates of how those effects limit the timescales of the superradiant laser?
- Comparing Fig. 2(a) and Fig. 4(a), it seems like the better overlap of atomic density and cavity mode in the trapped case does not strongly change the total steady-state photon number. Could the authors comment on why that is? And what sets the limit for the maximum number of photons in the cavity?

A few minor stylistic comments:
- The potential in the “Dynamics including an optical lattice potential” section is a bit hard to read. The authors might consider changing it from inline to paragraph.
- It would be beneficial to state explicitly the atom number used for the results of Fig. 3 and 4 in the caption of the respective figure.

Is the work clearly and accurately presented and does it cite the current literature?

Yes

Is the study design appropriate and does the work have academic merit?

Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Quantum optics, Cavity QED, Quantum phase transitions, Superradiance

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.